



Higher Mathematics

Trigonometry

Examples

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3 Solving Trigonometric Equations

RC

1. Solve $\sin x^\circ = \frac{1}{2}$ for $0 < x < 360$.

3 Solving Trigonometric Equations

RC



2. Solve $\cos x^\circ = -\frac{1}{\sqrt{5}}$ for $0 < x < 360$.

3 Solving Trigonometric Equations

RC

3. Solve $\sin x^\circ = 3$ for $0 < x < 360$.

3 Solving Trigonometric Equations

RC



4. Solve $\tan x^\circ = -5$ for $0 < x < 360$.

3 Solving Trigonometric Equations

RC

5. Solve $2\sin 2x^\circ - 1 = 0$ where $0 \leq x \leq 360$.

3 Solving Trigonometric Equations

RC

6. Solve $\sqrt{2} \cos 2x = 1$ where $0 \leq x \leq \pi$.

3 Solving Trigonometric Equations

RC

7. Solve $4\cos^2 x = 3$ where $0 < x < 2\pi$.

3 Solving Trigonometric Equations

RC



8. Solve $3 \tan(3x^\circ - 20^\circ) = 5$ where $0 \leq x \leq 360$.

3 Solving Trigonometric Equations

RC



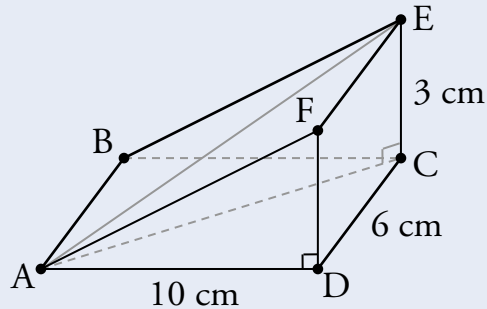
9. Solve $\cos\left(2x + \frac{\pi}{3}\right) = 0.812$ for $0 < x < 2\pi$.

4 Trigonometry in Three Dimensions

EF



1. The triangular prism ABCDEF is shown below.



Calculate the acute angle between:

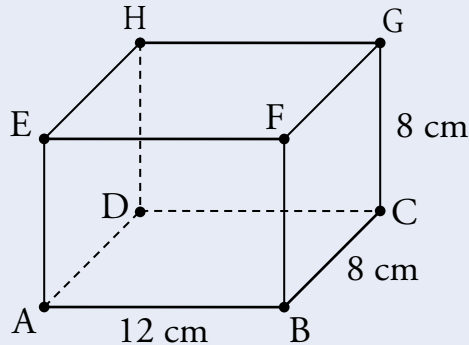
- The line AF and the plane ABCD.
- AE and ABCD.

4 Trigonometry in Three Dimensions

EF



2. ABCDEFGH is a cuboid with dimensions $12 \times 8 \times 8$ cm as shown below.



- Calculate the size of the angle between the planes AFGD and ABCD.
- Calculate the size of the acute angle between the diagonal planes AFGD and BCHE.

5 Compound Angles

EF

1. Expand and simplify $\cos(x^\circ + 60^\circ)$.

5 Compound Angles

EF

2. Show that $\sin(a + b) = \sin a \cos b + \cos a \sin b$ for $a = \frac{\pi}{6}$ and $b = \frac{\pi}{3}$.

5 Compound Angles

EF

3. Find the exact value of $\sin 75^\circ$.

5 Compound Angles

EF

Finding Trigonometric Ratios

4. Acute angles p and q are such that $\sin p = \frac{4}{5}$ and $\sin q = \frac{5}{13}$. Show that
- $$\sin(p+q) = \frac{63}{65}.$$

5 Compound Angles

EF

Using compound angle formulae to confirm identities

5. Show that $\sin\left(x - \frac{\pi}{2}\right) = -\cos x$.

5 Compound Angles

EF

Using compound angle formulae to confirm identities

6. Show that $\frac{\sin(s+t)}{\cos s \cos t} = \tan s + \tan t$ for $\cos s \neq 0$ and $\cos t \neq 0$.

6 Double-Angle Formulae

EF

1. Given that $\tan\theta = \frac{4}{3}$, where $0 < \theta < \frac{\pi}{2}$, find the exact value of $\sin 2\theta$ and $\cos 2\theta$.

6 Double-Angle Formulae

EF

2. Given that $\cos 2x = \frac{5}{13}$, where $0 < x < \pi$, find the exact values of $\sin x$ and $\cos x$.

7 Further Trigonometric Equations

RC

Solving equations involving $\sin 2x$ and either $\sin x$ or $\cos x$

1. Solve $\sin 2x^\circ = -\sin x^\circ$ for $0 \leq x < 360$.

7 Further Trigonometric Equations

RC

Solving equations involving $\cos 2x$ and $\cos x$

2. Solve $\cos 2x = \cos x$ for $0 \leq x \leq 2\pi$.

7 Further Trigonometric Equations

RC

Solving equations involving $\cos 2x$ and $\sin x$

3. Solve $\cos 2x = \sin x$ for $0 < x < 2\pi$.

8 Expressing $p\cos x + q\sin x$ in the form $k\cos(x - a)$


EF



1. Write $5\cos x^\circ + 12\sin x^\circ$ in the form $k\cos(x^\circ - a^\circ)$ where $0 \leq a < 360$.

8 Expressing $p\cos x + q\sin x$ in the form $k\cos(x - a)$

EF

-  2. Write $5\cos x - 3\sin x$ in the form $k\cos(x - a)$ where $0 \leq a < 2\pi$.

9 Expressing $p\cos x + q\sin x$ in other forms

EF



1. Write $4\cos x^\circ + 3\sin x^\circ$ in the form $k\sin(x^\circ + a^\circ)$ where $0 \leq a < 360$.

9 Expressing $p\cos x + q\sin x$ in other forms

EF



2. Write $\cos x - \sqrt{3} \sin x$ in the form $k \cos(x + a)$ where $0 \leq a < 2\pi$.

10 Multiple Angles

EF



Write $5 \cos 2x^\circ + 12 \sin 2x^\circ$ in the form $k \sin(2x^\circ + a^\circ)$ where $0 \leq a < 360$.

11 Maximum and Minimum Values

EF



Write $4 \sin x + \cos x$ in the form $k \cos(x - a)$ where $0 \leq a \leq 2\pi$ and state:

- (i) the maximum value and the value of $0 \leq x < 2\pi$ at which it occurs
- (ii) the minimum value and the value of $0 \leq x < 2\pi$ at which it occurs.

12 Solving Equations

RC



1. Solve $5 \cos x^\circ + \sin x^\circ = 2$ where $0 \leq x < 360$.

12 Solving Equations

RC



2. Solve $2 \cos 2x + 3 \sin 2x = 1$ where $0 \leq x < 2\pi$.

13 Sketching Graphs of $y = p\cos x + q\sin x$


EF



- (a) Write $7\cos x^\circ + 6\sin x^\circ$ in the form $k\cos(x^\circ - a^\circ)$, $0 \leq a < 360$.
(b) Hence sketch the graph of $y = 7\cos x^\circ + 6\sin x^\circ$ for $0 \leq x \leq 360$.

13 Sketching Graphs of $y = p\cos x + q\sin x$

EF

-  2. Sketch the graph of $y = \sin x^\circ + \sqrt{3} \cos x^\circ$ for $0 \leq x \leq 360$.

13 Sketching Graphs of $y = p\cos x + q\sin x$

EF



3. (a) Write $5\sin x^\circ - \sqrt{11}\cos x^\circ$ in the form $k\sin(x^\circ - a^\circ)$, $0 \leq a < 360$.
- (b) Hence sketch the graph of $y = 5\sin x^\circ - \sqrt{11}\cos x^\circ + 2$, $0 \leq x \leq 360$.