Higher

Mathematics

Differentiation

Examples

Contents

2 Finding the Derivative RC

1. Given $f(x) = x^4$, find $f'(x)$.

2 Finding the Derivative

RC

2. Differentiate $f(x) = x^{-3}$, $x \neq 0$, with respect to x.

2 Finding the Derivative

RC

3. Differentiate $y = x^{-\frac{1}{3}}$, $x \neq 0$, with respect to x.

2 Finding the Derivative RC

4. Find the derivative of $x^{\frac{3}{2}}$, $x \geq 0$, with respect to x.

2 Finding the Derivative RC

Preparing to differentiate

1. Differentiate $\sqrt{x}$ with respect to x, where $x > 0$.

2 Finding the Derivative RC

Preparing to differentiate

2. Given $y = \frac{1}{x^2}$, where $x \neq 0$, find $\frac{dy}{dx}$.

2 Finding the Derivative

RC

Terms with a coefficient

1. A function f is defined by $f(x)=2x^3$. Find $f'(x)$.

2 Finding the Derivative RC

Terms with a coefficient

2. Differentiate $y = 4x^{-2}$ with respect to x, where $x \neq 0$.

2 Finding the Derivative RC

Terms with a coefficient

3. Differentiate $\frac{2}{x^3}$, $x \neq 0$, with respect to x.

2 Finding the Derivative RC

Terms with a coefficient

4. Given $y = \frac{3}{2\sqrt{x}}$, $x > 0$, find $\frac{dy}{dx}$.

2 Finding the Derivative RC

Differentiating more than one term

1. A function f is defined for $x \in \mathbb{R}$ by $f(x) = 3x^3 - 2x^2 + 5x$.

   Find $f'(x)$.

2 Finding the Derivative RC

Differentiating more than one term

2. Differentiate $y = 2x^4 - 4x^3 + 3x^2 + 6x + 2$ with respect to x.

2 Finding the Derivative RC

Differentiating more complex expressions

1. Differentiate $y = \frac{1}{3x\sqrt{x}}$, $x > 0$, with respect to x.

2 Finding the Derivative RC

Differentiating more complex expressions

2. Find $\dfrac{dy}{dx}$ when $y = (x-3)(x+2)$.

2 Finding the Derivative RC

Differentiating more complex expressions

3. A function f is defined for $x \neq 0$ by $f(x)=\frac{x}{5}+\frac{1}{x^2}$. Find $f'(x)$.

2 Finding the Derivative

RC

Differentiating more complex expressions

4. Differentiate $\dfrac{x^4 - 3x^2}{5x}$ with respect to x, where $x \neq 0$.

2 Finding the Derivative RC

Differentiating more complex expressions

5. Differentiate $\dfrac{x^3 + 3x^2 - 6x}{\sqrt{x}}$, $x > 0$, with respect to x.

2 Finding the Derivative RC

Differentiating more complex expressions

6. Find the derivative of $y = \sqrt{x}\left(x^2 + \sqrt[3]{x}\right),\ x > 0$, with respect to x.

3 Differentiating with Respect to Other Variables RC

1. Differentiate $3t^2 - 2t$ with respect to t.

3 Differentiating with Respect to Other Variables RC

2. Given $A(r) = \pi r^2$, find $A'(r)$.

3 Differentiating with Respect to Other Variables RC

3. Differentiate px^2 with respect to p.

4 Rates of Change

1. Given $f(x) = 2x^5$, find the rate of change of f when $x = 3$.

4 Rates of Change RC

2. Given $y = \frac{1}{x^{\frac{2}{3}}}$ for $x \neq 0$, calculate the rate of change of y when $x = 8$.

4 Rates of Change

Displacement, velocity and acceleration

3. A ball is thrown so that its displacement s after t seconds is given by

   $s(t) = 12t - 5t^2$.

   Find its velocity after 2 seconds.

5 Equations of Tangents RC

1. Find the equation of the tangent to the curve with equation $y = x^2 - 3$ at the point $(2, 1)$.

5 Equations of Tangents RC

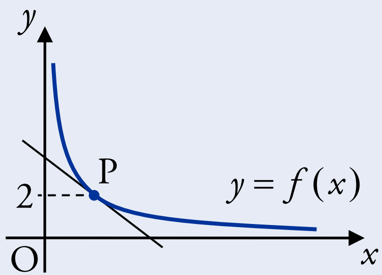
2. Find the equation of the tangent to the curve with equation $y = x^3 - 2x$ at the point where $x = -1$.

5 Equations of Tangents

RC

3. A function f is defined for $x > 0$ by $f(x) = \frac{1}{x}$.

   Find the equation of the tangent to the curve $y = f(x)$ at P.



5 Equations of Tangents RC

4. Find the equation of the tangent to the curve $y = \sqrt[3]{x}^{2}$ at the point where $x = -8$.

5 Equations of Tangents RC

5. A curve has equation $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x + 5$.

   Find the coordinates of the points on the curve where the tangent has gradient 4.

6 Increasing and Decreasing Curves RC

1. A curve has equation $y = 4x^2 + \frac{2}{\sqrt{x}}$.

   Determine whether the curve is increasing or decreasing at $x = 10$.

6 Increasing and Decreasing Curves RC

2. Show that the curve $y = \frac{1}{3}x^3 + x^2 + x - 4$ is never decreasing.

8 Determining the Nature of Stationary Points RC

1. A curve has equation $y = x^3 - 6x^2 + 9x - 4$.

   Find the stationary points on the curve and determine their nature.

8 Determining the Nature of Stationary Points RC

2. Find the stationary points of $y = 4x^3 - 2x^4$ and determine their nature.

8 Determining the Nature of Stationary Points RC

3. A curve has equation $y = 2x + \frac{1}{x}$ for $x \neq 0$. Find the x-coordinates of the stationary points on the curve and determine their nature.

9 Curve Sketching RC

Sketch the curve with equation $y = 2x^3 - 3x^2$.

10 Differentiating sin$x$ and cos$x$ RC

1. Differentiate $y = 3\sin x$ with respect to x.

10 Differentiating $\sin x$ and $\cos x$ RC

1. A function f is defined by $f(x) = \sin x - 2\cos x$ for $x \in \mathbb{R}$.

   Find $f'\left(\frac{\pi}{3}\right)$.

10 Differentiating sin*x* and cos*x* RC

2. A function f is defined by $f(x) = \sin x - 2\cos x$ for $x \in \mathbb{R}$.

   Find $f'\left(\frac{\pi}{3}\right)$.

10 Differentiating $\sin x$ and $\cos x$ RC

3. Find the equation of the tangent to the curve $y = \sin x$ when $x = \frac{\pi}{6}$.

11 The Chain Rule

RC

If $y = \cos\left(5x + \frac{\pi}{6}\right)$, find $\frac{dy}{dx}$.

12 Special Cases of the Chain Rule

RC

Powers of a Function

1. A function f is defined on a suitable domain by $f(x) = \sqrt{2x^2 + 3x}$.
   Find $f'(x)$.

12 Special Cases of the Chain Rule RC

Powers of a Function

2. Differentiate $y = 2\sin^4 x$ with respect to x.

12 Special Cases of the Chain Rule RC

Powers of a Linear Function

3. Differentiate $y = (5x + 2)^3$ with respect to x.

12 Special Cases of the Chain Rule

RC

Powers of a Linear Function

4. If $y = \dfrac{1}{(2x+6)^3}$, find $\dfrac{dy}{dx}$.

12 Special Cases of the Chain Rule

RC

Powers of a Linear Function

5. A function f is defined by $f(x) = \sqrt[3]{(3x-2)^4}$ for $x \in \mathbb{R}$. Find $f'(x)$.

12 Special Cases of the Chain Rule RC

Trigonometric Functions

6. Differentiate $y = \sin(9x + \pi)$ with respect to x.

13 Closed Intervals RC

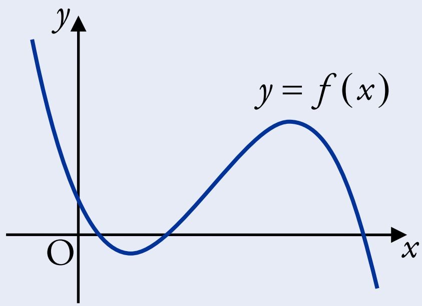
A function f is defined for $-1 \leq x \leq 4$ by $f(x) = 2x^3 - 5x^2 - 4x + 1$.

Find the maximum and minimum value of $f(x)$.

14 Graphs of Derivatives

EF

The curve $y = f(x)$ shown below is a cubic. It has stationary points where $x = 1$ and $x = 4$.

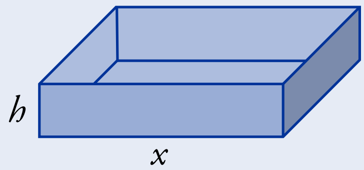


Sketch the graph of $y = f'(x)$.

15 Optimisation

A

1. Small wooden trays, with open tops and square bases, are being designed. They must have a volume of 108 cubic centimetres.



The internal length of one side of the base is x centimetres, and the internal height of the tray is h centimetres.

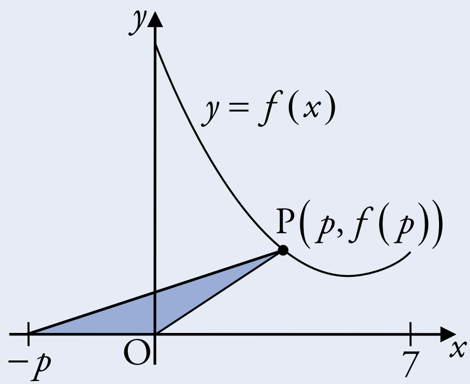
(a) Show that the total internal surface area A of one tray is given by

$$A = x^2 + \frac{432}{x}.$$

(b) Find the dimensions of the tray using the least amount of wood.

15 Optimisation A

2. The point P lies on the graph of $f(x)=x^2-12x+45$, between $x=0$ and $x=7$.



A triangle is formed with vertices at the origin, P and $(-p,0)$.

(a) Show that the area, A square units, of this triangle is given by

$$A=\frac{1}{2}p^3-6p^2+\frac{45}{2}p.$$

(b) Find the greatest possible value of A and the corresponding value of p for which it occurs.