Higher

Mathematics

UNIT 2

Specimen NAB Assessment

HSN22510

UNIT 2

Specimen NAB Assessment

Outcome 1

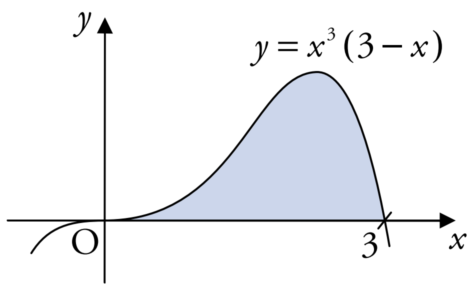
1. Show that $(x+2)$ is a factor of $f(x)=x^3-2x^2-4x+8$ and hence factorise fully $f(x)$. **4**

2. Use the discriminant to determine the nature of the roots of the equation $3x^2+4x-2=0$. **2**

Outcome 2

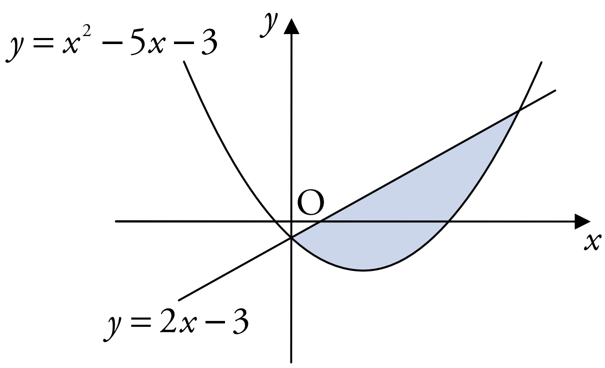
3. Find $\int \frac{6}{x^3}\,dx$, where $x \neq 0$. **3**

4. The curve $y=x^3(3-x)$ is shown in the diagram below.



Calculate the shaded area enclosed between the curve and the x-axis between $x=0$ and $x=3$. **5**

5. The diagram shows the line with equation $y=2x-3$ and the curve with equation $y=x^2-5x-3$.

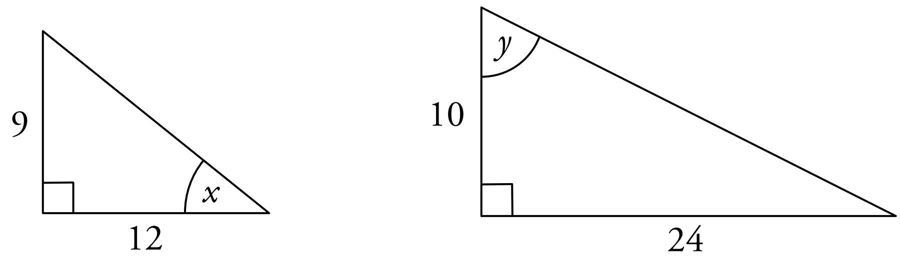


Write down the integral which represents the shaded area.
Do not carry out the integration. **3**

Outcome 3

6. Solve the equation $\sqrt{2}\sin 2x = 1$ for $0 \le x < \pi$. **3**

7. The diagram below shows two right-angled triangles.



   (a) Write down the values of $\sin x$ and $\cos y$. **2**

   (b) By expanding $\cos(x+y)$ show that the exact value of $\cos(x+y)$ is $-\frac{16}{65}$. **2**

8. (a) Express $\sin 15°\cos x° + \cos 15°\sin x°$ in the form $\sin(a° + b°)$. **1**

   (b) Use your answer from part (a) to solve the equation $\sin 15°\cos x° + \cos 15°\sin x° = \frac{\sqrt{3}}{2}$ for $0 < x < 360$. **4**

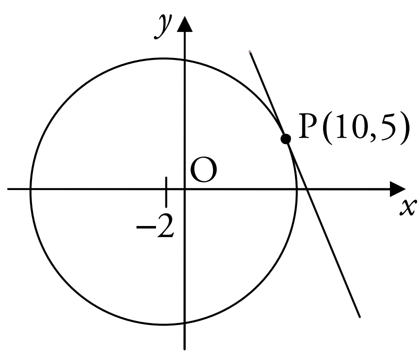
Outcome 4

9. (a) A circle has radius 7 units and centre $(2,-3)$.
   Write down the equation of the circle. **2**

   (b) A circle has equation $x^2 + y^2 - 10x + 6y - 3 = 0$.
   Write down its radius and the coordinates of its centre. **3**

10. Show that the straight line $y = -2x - 3$ is a tangent to the circle with equation $x^2 + y^2 + 6x + 4y + 8 = 0$. **5**

11. The point $P(10,5)$ lies on the circle with centre $(-2,0)$, as shown in the diagram below.



Find the equation of the tangent to the circle at P. **4**

Marking Instructions

Pass Marks

Outcome 1	Outcome 2	Outcome 3	Outcome 4
$\frac{4}{6}$	$\frac{8}{11}$	$\frac{7}{12}$	$\frac{10}{14}$

Outcome 1 – Polynomials and Quadratics

1.

$$\begin{array}{r|rrrr} -2\checkmark & 1 & -2 & -4 & 8 \\ & & -2 & 8 & -8 \\ \hline & 1 & -4 & 4 & 0 \end{array}$$

Since $f(-2)=0$, $(x+2)$ is a factor. ✓

$f(x)=(x+2)(x-4x+4)$ ✓

$=(x+2)(x-2)(x-2)$ ✓

- Know to evaluate $f(-2)$
- Complete evaluation and conclusion
- Quadratic factor
- Factorise quadratic

4

2. b^2-4ac ✓

$=4^2-4\times3\times(-2)$

$=40$

Since $b^2-4ac>0$, the roots are real and distinct. ✓

- Use the discriminant
- Calculate discriminant and state nature of roots

2

Outcome 2 – Integration

3. $\int \frac{6}{x^3}\,dx=\int(6x^{-3})\,dx$ ✓

$=\frac{6x^{-2}}{-2}+c$

$=-3x^{-2}$ ✓ $+c$ ✓

- Express in standard form
- Integrate term with negative power
- Constant of integration

3

4. $\int_0^3$ ✓ $x^3(3-x)\,dx=\int_0^{3}$ ✓ $(3x^3-x^4)\,dx$

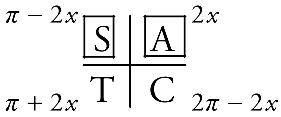
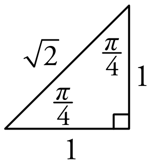
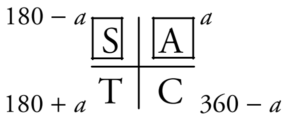
$=\left[\frac{3x^4}{4}-\frac{x^5}{5}\right]_0^3$ ✓

$=\left(\frac{3}{4}(3)^4-\frac{1}{5}(3)^5\right)-0$ ✓

$=\frac{243}{20}$ ✓ $\left(\text{or } 12\frac{3}{20}\right)$

- Know to integrate with limits
- Use correct limits
- Integrate
- Process limits
- Complete process

5

5. $x^2-5x-3=2x-3$ ✓ $x^2-7x=0$ $x(x-7)=0$ $x=0$ or $x=7$ ✓ Shaded area is $\int_0^7 \left((2x-3)-(x^2-5x-3)\right)dx$ ✓ square units.	• Strategy to find intersection • Solve quadratic • Use $\int(\text{upper}-\text{lower})\,dx$ with limits from quadratic	**3**
Outcome 3 – Trigonometry		
6. $\sin 2x=\frac{1}{\sqrt{2}}$ ✓   $2x=\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4}$ $2x=\frac{\pi}{4}$ or $\pi-\frac{\pi}{4}$ $x=\frac{\pi}{8}$✓ or $\frac{3\pi}{8}$✓	• Rearrange to standard form • One solution • Second solution	**3**
7. (a) $AC=\sqrt{9^2+12^2}=15$, $DF=\sqrt{10^2+24^2}=26$ ✓ $\sin x=\frac{9}{15}=\frac{3}{5}$ and $\cos x=\frac{10}{26}=\frac{5}{13}$ ✓	• Calculate remaining sides • $\sin x$ and $\cos y$	**2**
(b) $\cos(x+y)=\cos x\cos y-\sin x\sin y$ ✓ $=\frac{4}{5}\times\frac{5}{13}-\frac{3}{5}\times\frac{12}{13}$ ✓ $=\frac{20}{65}-\frac{36}{65}$ $=-\frac{16}{65}$	• Use compound angle formula • Substitute values	**2**
8. (a) $\sin 15°\cos x°+\cos 15°\sin x°=\sin(15°+x°)$ ✓	• Use compound angle formula	**1**
(b) $\sin(15°+x°)=\frac{\sqrt{3}}{2}$✓  $a=\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)=60$ ✓ $15+x=60$ or $180-60$ $x=45$✓ or 105✓	• Substitute $\sin(15°+x°)$ • Process $\sin^{-1}$ • One solution • Second solution	**4**

Outcome 4 – Circles		
9. (a) $(x-2)^2+(y+3)^2$ ✓ $=49$ ✓	• Centre • Square of radius	**2**
(b) The centre is $(5,-3)$ ✓ The radius is $\sqrt{(-5)^2+3^2-(-3)}$ ✓ $=\sqrt{37}$ ✓	• State centre • Know how to calculate radius • Process radius	**3**
10. $x^2+y^2+6x+4y+8=0$ $x^2+(-2x-3)^2+6x+4(-2x-3)+8=0$ ✓ $5x^2+10x+5=0$ ✓ $x^2+2x+1=0$ b^2-4ac ✓ $=2^2-4\times1\times1$ $=16-16$ $=0$ ✓ Since the discriminant is zero, the line is a tangent to the circle. ✓	• Strategy for finding intersection • Express in standard form • Know to calculate discriminant • Calculate discriminant • Conclusion	**5**
11. $m_{\text{PC}}=\dfrac{5-0}{10+2}$ ✓ $=\frac{5}{12}$ ✓ So $m_{\text{tgt}}=-\frac{12}{5}$ ✓ since the radius and tangent are perpendicular. $y-5=-\frac{12}{5}(x-10)$ ✓ $12x+5y-145=0$	• Know how to find gradient of radius • Process gradient of radius • Gradient of tangent • Equation of tangent	**4**