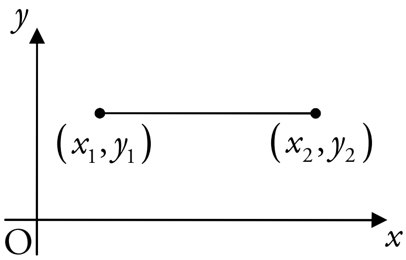
OUTCOME 1

Straight Lines

1 The Distance Between Points

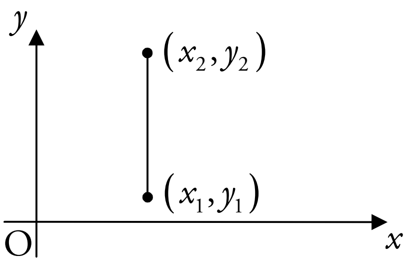
Points on Horizontal or Vertical Lines

It is relatively straightforward to work out the distance between two points which lie on a line parallel to the x- or y-axis.



In the diagram to the left, the points (x_1, y_1) and (x_2, y_2) lie on a line parallel to the x-axis, i.e. $y_1 = y_2$.

The distance between the points is simply the difference in the x-coordinates, i.e. $x_2 - x_1$ where $x_2 > x_1$.



In the diagram to the left, the points (x_1, y_1) and (x_2, y_2) lie on a line parallel to the y-axis, i.e. $x_1 = x_2$.

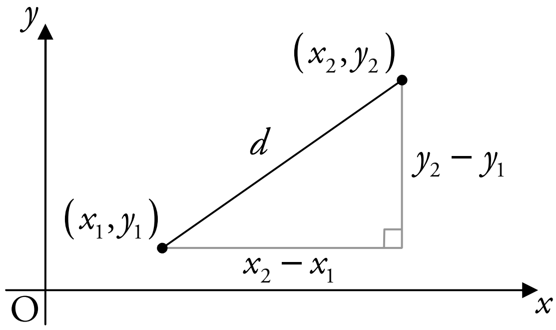
The distance between the points is simply the difference in the y-coordinates, i.e. $y_2 - y_1$ where $y_2 > y_1$.

EXAMPLE

1. Calculate the distance between the points $(-7, -3)$ and $(16, -3)$.

The Distance Formula

The distance formula gives us a method for working out the length of the straight line between **any** two points. It is based on Pythagoras's Theorem.



The distance d between the points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ units}$$

EXAMPLES

2. A is the point $(-2, 4)$ and $B(3, 1)$. Calculate the length of the line AB.

3. Calculate the distance between the points $\left(\frac{1}{2}, -\frac{15}{4}\right)$ and $(-1, -1)$.

2 The Midpoint Formula

The point half-way between two points is called the midpoint. It is calculated as follows:

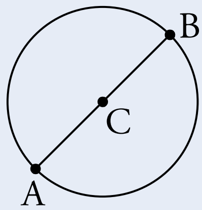
The midpoint M between (x_1, y_1) and (x_2, y_2) is

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

EXAMPLES

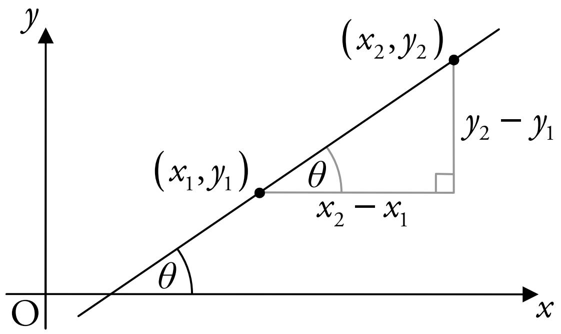
1. Calculate the midpoint of the points $(1, -4)$ and $(7, 8)$.

2. In the diagram below, $A(9, -2)$ lies on the circumference of the circle with centre $C(17, 12)$, and the line AB is the diameter of the circle. Find the coordinates of B.



3 Gradients

Consider a straight line passing through the points (x_1, y_1) and (x_2, y_2):



The gradient m of the line through (x_1, y_1) and (x_2, y_2) is:

$$m = \frac{\text{change in vertical height}}{\text{change in horizontal distance}} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{for } x_1 \neq x_2$$

Also, since $\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{y_2 - y_1}{x_2 - x_1}$ we obtain:

$$m = \tan\theta$$

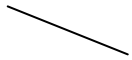
where θ is the angle between the line and the positive direction of the x-axis.

Note

As a result of the above definitions:

- lines with positive gradients slope up, from left to right

- lines with negative gradients slope down, from left to right



- lines parallel to the x-axis have a gradient of zero

- lines parallel to the y-axis have an undefined gradient

We may also use the fact that:

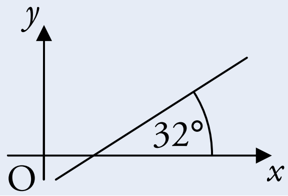
Lines are parallel $\Leftrightarrow$ they have the same gradient.

Note

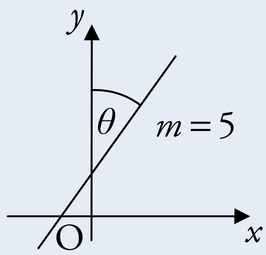
A $\Leftrightarrow$ B means:
If A is true, then B is true
and
If B is true, then A is true

EXAMPLES

1. Calculate the gradient of the straight line shown in the diagram below.



2. Find the size of angle θ shown in the diagram below.



3. Find the angle that the line joining $\mathrm{P}(-2, -2)$ and $\mathrm{Q}(1, 7)$ makes with the positive direction of the x-axis.