Mathematics and Physics

Useful Formulae

1 Pure Mathematics

1.1 Algebra

Fundamental Theorem of Algebra

$P(z)$ has n roots, such that each root is in the complex plane, if $P(z)$ has degree n. For a quadratic, the roots are:

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In a quadratic, if the roots are α and β, then

$$\alpha + \beta = -\frac{b}{a} \qquad \alpha\beta = \frac{c}{a}$$

De Moivre's Theorem

$$\forall n \in \mathbb{Q}, \qquad \begin{aligned} (a+bi)^n &= [r(\cos\theta + i\sin\theta)]^n \\ &= r^n(\cos n\theta + i \sin n\theta) \end{aligned}$$

Properties of Complex Numbers

$\forall z, w \in \mathbb{C}$,

$$\begin{aligned} \bar{\bar{z}} &= z \\ z &= \bar{z} \text{ if } z \in \mathbb{R} \\ z + \bar{z} &= 2\text{Re}(z) \\ |\bar{z}| &= |z| \\ z\bar{z} &= |z|^2 \\ \overline{(z+w)} &= \bar{z} + \bar{w} \\ \overline{(zw)} &= \bar{z}\bar{w} \\ \text{Re}(z) &< |z| \\ |z+w| &\leq |z| + |w| \end{aligned}$$

Binomial Theorem

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r \ , \ n \in \mathbb{N}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ \dots \ , \ n \in \mathbb{Q}$$

1.2 Trigonometric Identities

$$\begin{aligned} \sin(\alpha \pm \beta) &= \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \\ \cos(\alpha \pm \beta) &= \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \\ \tan(\alpha \pm \beta) &= \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta} \end{aligned}$$

Table 1 Standard Integrals and Derivatives

$f(x)$	$f'(x)$	$\int f(x)\,dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1} + C$
$\sin x$	$\cos x$	$-\cos x + C$
$\cos x$	-$\sin x$	$\sin x + C$
$\tan x$	$\sec^2 x$	$\ln\lvert\sec x\rvert + C$
$\sec^2 x$		$\tan x + C$
$\sec x$	$\sec x \tan x$	$\ln\lvert\sec x + \tan x\rvert + C$
$\text{cosec}\, x$	$-\text{cosec}\, x \cot x$	$\ln\left\lvert\tan \frac{1}{2}x\right\rvert + C$
$\cot x$	$-\text{cosec}^2 x$	$\ln\lvert\sin x\rvert + C$
e^x	e^x	$e^x + C$
$\ln x$	$\frac{1}{x}$	
$\frac{1}{x}$	$-\frac{1}{x^2}$	$\ln\lvert x\rvert + C$
$f(g)$	$f'(g)g'$	
fg	$f'g + fg'$	
$\frac{f}{g}$	$\frac{f'g - fg'}{g^2}$	
$\sin^{-1}\left(\frac{x}{a}\right)$	$\frac{1}{\sqrt{a^2 - x^2}}$	
$\cos^{-1}\left(\frac{x}{a}\right)$	$\frac{-1}{\sqrt{a^2 - x^2}}$	
$\tan^{-1}\left(\frac{x}{a}\right)$	$\frac{a}{a^2 + x^2}$	
$\frac{1}{a^2 + x^2}$		$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

$$\begin{aligned} 2\sin\alpha\cos\beta &\equiv \sin(\alpha+\beta) + \sin(\alpha-\beta) \\ 2\cos\alpha\sin\beta &\equiv \sin(\alpha+\beta) - \sin(\alpha-\beta) \\ 2\cos\alpha\cos\beta &\equiv \cos(\alpha+\beta) + \cos(\alpha-\beta) \\ -2\sin\alpha\sin\beta &\equiv \cos(\alpha+\beta) - \cos(\alpha-\beta) \end{aligned}$$

$$\begin{aligned} \sin\theta + \sin\phi &\equiv 2\sin\tfrac{1}{2}(\theta+\phi)\cos\tfrac{1}{2}(\theta-\phi) \\ \sin\theta - \sin\phi &\equiv 2\cos\tfrac{1}{2}(\theta+\phi)\sin\tfrac{1}{2}(\theta-\phi) \\ \cos\theta + \cos\phi &\equiv 2\cos\tfrac{1}{2}(\theta+\phi)\cos\tfrac{1}{2}(\theta-\phi) \\ \cos\theta - \cos\phi &\equiv -2\sin\tfrac{1}{2}(\theta+\phi)\sin\tfrac{1}{2}(\theta-\phi) \end{aligned}$$

$$\begin{aligned} \sin 2\alpha &= 2\sin\alpha\cos\alpha \\ \cos 2\alpha &= \cos^2\alpha - \sin^2\alpha \\ &= 2\cos^2\alpha - 1 \\ &= 1 - 2\sin^2\alpha \\ \tan 2\alpha &= \frac{2\tan\alpha}{1 - \tan^2\alpha} \end{aligned}$$

Table 2 Values of Common Functions

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	x	$\log_a x$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	undefined
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	1	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	-	a	1

$$\begin{aligned}
\sin^2\alpha &= \tfrac{1}{2}(1-\cos 2\alpha) \\
&= \tfrac{1}{2}(3-\sin 2\alpha) \\
&= 1-\cos^2\alpha \\
\cos^2\alpha &= \tfrac{1}{2}(\sin 2\alpha - 1) \\
&= \tfrac{1}{2}(1+\cos 2\alpha) \\
\sec^2\alpha &= 1+\tan^2\alpha \\
\mathrm{cosec}^2\alpha &= 1+\cot^2\alpha
\end{aligned}$$

$$\begin{aligned}
\sin 3\alpha &= 3\sin\alpha - 4\sin^3\alpha \\
\cos 3\alpha &= 4\cos^3\alpha - 3\cos\alpha \\
\tan 3\alpha &= \frac{3\tan\alpha - \tan^3\alpha}{1-3\tan^2\alpha}
\end{aligned}$$

Hyperbolic Functions

$$\begin{aligned}
\sinh(x) &= \frac{e^x - e^{-x}}{2} = -i\sin(ix) \\
\cosh(x) &= \frac{e^x + e^{-x}}{2} = \cos(ix) \\
\tanh(x) &= \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{\sinh(x)}{\cosh(x)}
\end{aligned}$$

1.3 Vectors

Magnitude of Vector **a**:

$$|\mathbf{a}| = \sqrt{p^2+q^2+r^2}$$

where $\mathbf{a} = p\mathbf{i} + q\mathbf{j} + r\mathbf{k}$.

Unit Vector:

$$\hat{\mathbf{a}} = \frac{\mathbf{a}}{|\mathbf{a}|}$$

Scalar Product:

$$\begin{aligned}
\mathbf{a}\cdot\mathbf{b} &= |\mathbf{a}|\,|\mathbf{b}|\cos\theta \\
(x_1,y_1,z_1)\cdot(x_2,y_2,z_2) &= x_1x_2 + y_1y_2 + z_1z_2
\end{aligned}$$

Vector Product

For two vectors **a** and **b**, the vector product is notated as

$$\mathbf{a}\wedge\mathbf{b}$$

and is calculated from

$$(a_2b_3 - a_3b_2)\mathbf{i} - (a_1b_3 - a_3b_1)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}$$

It can also be obtained from:

$$\mathbf{a}\wedge\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\hat{\mathbf{u}}$$

where $\hat{\mathbf{u}}$ is the unit vector perpendicular to the plane of **a** and **b**. The Vector Product is the determinant of

$$\begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix}$$

Scalar Triple Product:

$$\mathbf{a}\cdot(\mathbf{b}\wedge\mathbf{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

1.4 Equations of Lines and Planes

Vector equation of a line:

$$\mathbf{r} = \mathbf{a} + \lambda\mathbf{d}$$

where **a** is a known point on the line, and **d** is a direction vector.

Parametric equations of a line:

$$\begin{aligned}
x &= a_i + \lambda d_i \\
y &= a_j + \lambda d_j \\
z &= a_k + \lambda d_k
\end{aligned}$$

Symmetric form of the equation of a line:

$$\frac{x-a_i}{d_i} = \frac{y-a_j}{d_j} = \frac{z-a_k}{d_k} = \lambda$$

Vector equation of a plane:

$$\mathbf{r}\cdot\mathbf{n} = \mathbf{a}\cdot\mathbf{n}$$

where **n** is the normal to the plane, and **a** is a known point on the plane.

Cartesian form of the equation of a plane, for a normal vector $n_1\mathbf{i} + n_2\mathbf{j} + n_2\mathbf{k}$:

$$xn_1 + yn_2 + xn_3 = d$$

where $d = \mathbf{a}\cdot\mathbf{n}$, and **a** is the position vector of a known point on the plane.

Parametric form of equation of plane:

$$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$$

where **a** is the position vector of a point on the plane, and **b** and **c** are non parallel vectors in the plane.

1.5 Matrices

Determinant of 2×2 matrix:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Determinant of 3×3 matrix:

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

Inverse of a matrix:

$$A^{-1} = \frac{1}{\mathrm{Det}(A)} \cdot \mathrm{Adj}(A)$$

where $\mathrm{Adj}(A)$ returns the adjugate matrix, the transpose of the matrix of cofactors. For a 2×2 matrix, the inverse is given by:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{\mathrm{Det}A} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

1.6 Geometry

Circles

Equation of a circle, centre (a, b) and radius r:

$$(x-a)^2 + (y-b)^2 = r^2$$

General Equation of a Circle:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

where the centre is $(-g, -f)$ and the radius is

$$\sqrt{g^2 + f^2 - c}$$

Cosine Rule

In a triangle ABC:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2r$$

where r is the radius of the circumcircle of $\triangle$ABC.

1.7 Laws of Logarithms

$$\log_a xy = \log_a x + \log_a y$$
$$\log_a \frac{x}{y} = \log_a x - \log_a y$$
$$\log_a x^n = n \log_a x$$

1.8 Sequences and Series

An arithmetic sequence is one of the form:

$$a\ ,\ a+d\ ,\ a+2d\ ,\ \ldots\ ,\ a+(n-1)d\ ,\ \ldots$$

where d is the common difference. A geometric sequence takes the form

$$a\ ,\ ar\ ,\ ar^2\ ,\ \ldots\ ,\ ar^{n-1}\ ,\ \ldots$$

where r is the common ratio $\dfrac{U_n}{U_{n-1}}$.

Sum to n terms of an arithmetic series:

$$S_n = \tfrac{1}{2}n\,[2a + (n-1)d]$$

Sum to n terms of a geometric series:

$$S_n = \frac{a(1-r^n)}{1-r}\ ,\ n \in \mathbb{N}$$

where a is the first term in the sequence. Also,

$$S_\infty = \frac{a}{1-r} \quad \text{if } |r| < 1$$

General Expansions

$$\frac{1}{1-x} = 1 + x + x^2 + x^3\ \ldots\ ,\ |x| = 1$$
$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$$
$$e^{ix} = \cos x + i \sin x$$
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \ldots$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \ldots$$
$$\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e$$
$$\lim_{n\to\infty} \left[n \tan\left(\frac{180}{n}\right)\right] = \pi$$
$$\sum_{r=1}^{n} r = \tfrac{1}{2}n(n+1)\ ,\ n \in \mathbb{N}$$
$$\sum_{r=0}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)$$

MacLaurin Expansion

The MacLaurin series for a function $f(x)$ is

$$\begin{aligned} f(x) &= f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + \ldots + f^{(n)}(0)\frac{x^n}{n!} \\ &= \sum_{r=0}^{n} f^{(r)}(0)\frac{x^r}{r!} \end{aligned}$$

1.9 Differential Equations

First Order, Linear

An equation of the form

$$\frac{dy}{dx} + P(x)y = f(x)$$

can be solved using

$$e^{\int P(x)\,dx} y = \int e^{\int P(x)\,dx} f(x)\,dx$$

Second Order, Linear, Homogeneous

For an equation

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$

there is an auxiliary equation

$$a\omega^2 + b\omega + c = 0$$

- For real and distinct roots for ω

$$y = Ae^{\omega_1 x} + Be^{\omega_2 x}$$

- For real and equal roots for ω

$$y = (A + Bx)e^{\omega x}$$

- For complex roots $p \pm iq$

$$y = e^{px}(A\cos qx + B\sin qx)$$

2 Greek Alphabet

See Table 4 on Page 10.

3 Applied Mathematics: Mechanics

3.1 Basic Kinematics

Let displacement equal s, velocity equal v, and acceleration equal a. Alternatively, displacement is x, velocity $\dot{x}$, and acceleration $\ddot{x}$.

$$\begin{aligned} v &= \frac{ds}{dt} = \frac{d}{dt}\left(ut + \tfrac{1}{2}at^2\right) \\ a &= \frac{dv}{dt} = \frac{d}{dt}\,(u + at) \\ a &= \frac{dv}{dx}\cdot\frac{dx}{dt} = v\frac{dv}{dx} \end{aligned}$$

3.2 Projectiles

For a projectile, the displacement **r**, in two dimensions, is defined as:

$$\mathbf{r} = \left(ut\cos\alpha\,,\ ut\sin\alpha - \tfrac{1}{2}gt^2\right)$$

where **v**, the velocity, it defined as

$$\mathbf{v} = (u\cos\alpha\,,\ u\sin\alpha - gt)$$

An equation in x and y for the trajectory is given as:

$$\begin{aligned} y &= x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha} \\ &= x\tan\alpha - \frac{g}{2u^2}\,x^2\,\sec^2\alpha \end{aligned}$$

The time of flight of a projectile:

$$t = \frac{2u\sin\alpha}{g}$$

The horizontal range for a horizontal plane:

$$x = \frac{u^2\sin 2\alpha}{g}$$

Maximum height of projectile:

$$y = \frac{u^2\sin^2\alpha}{2g}$$

3.3 Friction

For an object on the point of slipping:

$$F = \mu R$$

where μ is the coefficient of friction. The coefficient of friction is given by:

$$\mu = \tan\lambda$$

where λ is the angle of friction. (i.e. The angle of the slope on the point of the object slipping.)

3.4 Oscillations & S.H.M.

Hooke's Law:

$$T = kx = \frac{\lambda x}{l}$$

where k is the stiffness constant, l the natural length, and λ the modulus of elasticity.

Equations for S.H.M.:

$$\begin{aligned} \ddot{x} &= -\omega^2 x \\ v^2 &= \omega^2(a^2 - x^2) \\ x &= a\sin(\omega t + \alpha) \end{aligned}$$

where a is the amplitude and α is the phase of the oscillations.

Maximum Speed and Acceleration:

$$v_{max} = a\omega$$
$$\ddot{x}_{max} = a\omega^2$$

Period:

$$\tau = \frac{2\pi}{\omega}$$

Damped Oscillations

Equation of motion for a damped oscillating system:

$$m\ddot{x} + c\dot{x} + kx = 0$$

Basic form of solution:

$$x = Ae^{\omega t}$$

Auxilliary Equation for $[m\ddot{x} + c\dot{x} + kx = 0]$:

$$m\omega^2 + c\omega + k = 0$$

Solutions to Auxilliary Equation:

- If $c^2 > 4mk$ then the negative roots are ω_1 and ω_2, giving:
$$x = Ae^{\omega_1 t} + Be^{\omega_2 t}$$
- If $c^2 = 4mk$ then the roots are equal, giving:
$$x = (A + Bt)e^{\omega t}$$
where $\omega = -\frac{c}{2m}$
- If $c^2 < 4mk$ the roots are imaginary:
$$\omega = -\frac{c}{2m} \pm in$$
where $i = \sqrt{-1}$ and $n = \sqrt{\frac{4mk - c^2}{4m^2}}$

Final Equation of Motion:

$$\begin{aligned} x &= Ce^{-\frac{c}{2m}t}\cos(nt + \phi) \\ &= e^{-\frac{c}{2m}t}[C\cos(nt) + D\sin(nt)] \end{aligned}$$

The trigonometric functions are the result of an infinite expansion for e^x.

3.5 Circular Motion

Unit Vectors:

$$\begin{aligned} \mathbf{e}_r &= \cos\theta\mathbf{i} + \sin\theta\mathbf{j} \\ \mathbf{e}_\theta &= -\sin\theta\mathbf{i} + \cos\theta\mathbf{j} \end{aligned}$$

Displacement of particle moving with Circular Motion:

$$\begin{aligned} \mathbf{r} &= r\cos\theta\mathbf{i} + r\sin\theta\mathbf{j} \\ &= r\mathbf{e}_r \end{aligned}$$

Velocity of particle:

$$\begin{aligned} \mathbf{v} &= -r\sin\theta\frac{d\theta}{dt}\mathbf{i} + r\cos\theta\frac{d\theta}{dt}\mathbf{j} \\ &= r\frac{d\theta}{dt}\mathbf{e}_\theta \end{aligned}$$

Acceleration of Particle:

$$\mathbf{a} = -r\left(\frac{d\theta}{dt}\right)^2\mathbf{e}_r + r\frac{d^2\theta}{dt^2}\mathbf{e}_\theta$$

Where $\frac{d\theta}{dt}$ is constant

$$\begin{aligned} \mathbf{r} &= r\cos(\omega t)\mathbf{i} + r\sin(\omega t)\mathbf{j} \\ \mathbf{v} &= r\omega[-\sin(\omega t)\mathbf{i} + \cos(\omega t)\mathbf{j}] \\ \mathbf{a} &= -r\omega^2[\cos(\omega t)\mathbf{i} + \sin(\omega t)\mathbf{j}] = -\omega^2\mathbf{r} \end{aligned}$$

3.6 Dynamics

The Impulse (and change in momentum) of a force is given by:

$$\int_{t_1}^{t_2} \mathbf{F}\,dt = m\mathbf{v} - m\mathbf{u}$$

Work done for a constant force:

$$W = \mathbf{F}\cdot\mathbf{d}$$

Work done for a force as a function of x or t:

$$\begin{aligned} W &= \int_{x_1}^{x_2} \mathbf{F}(x)dx \\ &= \int_{t_1}^{t_2} \mathbf{F}(t)\cdot\mathbf{v}dt \quad \text{where } \mathbf{v} = \frac{dx}{dt}\mathbf{i} \end{aligned}$$

Power is given by:

$$P = \frac{dW}{dt} = \mathbf{F}\cdot\mathbf{v}$$

4 Physics: Mechanics

4.1 Angular Motion

In angular motion, the angular displacement is represented by θ, angular velocity ω and angular acceleration α. The following table presents angular motion equations as they are analogous to linear motion equations:

Linear	**Angular**
$s = ut + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v = u + at$	$\omega = \omega_0 + \alpha t$

The period and frequency of an object's rotation:

$$\begin{aligned} T &= \frac{2\pi}{\omega} \\ f &= \frac{1}{T} = \frac{\omega}{2\pi} \end{aligned}$$

Families of angular movement equations:

Tangential Velocity:

$$v = \frac{r\theta}{t} = r\omega$$

Tangential Acceleration:

$$a_{tan} = r\alpha$$

Centripetal Acceleration:

$$a_{cent} = \frac{v^2}{r} = r\omega^2$$

Centripetal Force:

$$F_{cent} = \frac{mv^2}{r} = mr\omega^2$$

4.2 Rotational Dynamics

Torque on an object:

$$T = Fr = I\alpha = mr^2\alpha$$

Moment of Inertia:

$$I = \sum_{i=1}^{n} m_i r_i^2$$

Angular Momentum L:

$$L = I\omega$$

Rotational Kinetic Energy:

$$E_{rk} = \tfrac{1}{2} I\omega^2$$

Work Done:

$$E_W = T\theta$$

4.3 Relativistic Dynamics

$$E = mc^2$$
$$mc^2 = T + m_0c^2$$

Relativistic Mass at velocity v:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

4.4 Simple Harmonic Motion

Simple Harmonic Motion Equation:

$$\frac{d^2y}{dt^2} = -\omega^2 y$$

Solutions to the S.H.M Equation:

$$\begin{aligned} y &= A\sin\omega t \\ &= A\cos\omega t \end{aligned}$$

where A is the maximum displacement (amplitude).

Velocity of the particle:

$$v = \pm\omega\sqrt{A^2 - y^2}$$

Kinetic Energy in S.H.M.:

$$E_K = \tfrac{1}{2} m\omega^2 (A^2 - y^2)$$

Potential Energy:

$$E_P = \tfrac{1}{2} m\omega^2 y^2$$

4.5 Gravitation

Force between two masses:

$$F = \frac{Gm_1m_2}{r^2}$$

Gravitational Potential:

$$V = -\frac{Gm}{r}$$

Gravitational Potential Energy:

$$E_p = -\frac{Gm_1m_2}{r}$$

Gravitational Field Strength:

$$g = \frac{Gm}{r^2}$$

4.6 Wave-Particle Duality

Quantisation of Angular Momentum:

$$mvr = n\hbar$$

De Broglie Wavelength:

$$\lambda = \frac{h}{p}$$

5 Physics: Electrical Phenomena

Any E without a subscript refers to the electric field strength, where as a subscript E refers to an energy value. (This applies to Section 5 only.)

5.1 Electric Fields

Force exerted on a charge q, in a field of strength E:

$$F = qE$$

Force between two charges Q_1 and Q_2, r metres apart:

$$|F| = \frac{Q_1Q_2}{4\pi\epsilon_0 r^2}$$

where ϵ_0 is the permitivity of free space.

Electric field strength in uniform field between two plates, d metres apart:

$$E = \frac{V}{d}$$

Electric field strength r metres from charge Q:

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

Electrostatic Potential, r metres from charge Q:

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

Electrostatic Potential Energy:

$$E_{ESP} = qV$$

Final Velocity of particle moving through uniform field:

$$v = \sqrt{\frac{2qV}{m}}$$

Distance of closest approach of charge q:

$$r = \frac{2qQ}{4\pi\epsilon_0 m v^2}$$

Acceleration of particle in uniform field:

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{qV}{md}$$

5.2 Magnetic Fields

Force on current-carrying conductor in a magnetic field:

$$F = BIl \sin\theta = qvB \sin\theta$$

Magnetic induction due to a conductor:

$$B = \frac{\mu_0 I}{2\pi r}$$

where μ_0 is the permeability of free space.

Force between two wires:

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

For a charged particle in a magnetic field:

$$qvB = \frac{mv^2}{r}$$

where r is the radius of the circle in which the particle is moving.

For a particle moving in a magnetic field:

$$d = \frac{2\pi r}{\tan\theta}, \; 0 < \theta < \frac{\pi}{2}$$

where d is the pitch of the Helix.

5.3 Electromagnetic Fields

Velocity of a charge moving perpendicular to E and B fields:

$$v = \frac{E}{B}$$

Relating important physical constants:

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

5.4 Self Inductance

E.M.F. of an inductor in a circuit:

$$\epsilon = -L\frac{dI}{dt}$$

Energy in the magnetic field in the inductor:

$$E_E = \tfrac{1}{2} L I^2$$

Reactance of an inductor:

$$X_L = \frac{V_0}{I_0} = 2\pi f L$$

6 Physics: Wave Phenomena

Equation of a travelling wave:

$$y(x,t) = A \sin 2\pi \left(ft - \frac{x}{\lambda} \right)$$

where λ is the wavelength, f the frequency and A the amplitude.

Phase difference ϕ:

$$\phi = \frac{2\pi x}{\lambda}$$

where x is the displacement of one wave relative to the other.

Wavelength of nth harmonic:

$$\lambda_n = \frac{\lambda_1}{n}$$

nth harmonic frequency:

$$f_n = \frac{nv}{\lambda_1} = nf_1$$

The Doppler Effect

For a source moving towards a stationary observer:

$$f' = f\frac{v}{v - v_s}$$

For a source moving away from a stationary observer:

$$f' = f\frac{v}{v+v_s}$$

For an observer moving towards a stationary source:

$$f' = f\frac{v+v_{obs}}{v}$$

For an observer moving away from a stationary source:

$$f' = f\frac{v-v_{obs}}{v}$$

6.1 Interference by Division of Amplitude

The optical path length is defined as nd, where d is the distance travelled, and n the refractive index of the material. For constructive interference,

$$\text{OPD} = m\lambda \;,\; m \in \mathbb{Z}$$

For destructive interference,

$$d = \left(m + \tfrac{1}{2}\right)\lambda$$

For an arrangement such that $n_1 < n_3 < n_2$,

$$d = \frac{\left(m - \tfrac{1}{2}\right)\lambda}{2n}$$

where d is the thickness of the film and $m \in \mathbb{Z}$. For destructive interference, as in the above situation,

$$d = \frac{m\lambda}{2n}$$

For a lens coating,

$$d = \frac{\left(m + \tfrac{1}{2}\right)\lambda}{2n}$$

The minimum thickness of a lens coating to reduce glare is given by

$$d = \frac{\lambda}{4n}$$

Fringe seperation in wedge fringes:

$$\Delta x = \frac{\lambda l}{2ny}$$

where n is the refractive index of the material between the slides.

Fringe seperation in Young's Slits:

$$\Delta x = \frac{\lambda D}{d}$$

Malus' Law for polarisation:

$$I = I_0 \cos^2\theta$$

Brewster's Law for polarisation by reflection:

$$\tan i_p = n$$

where n is the refractive index of the reflective material.

7 Physics: Uncertainties

Calculation	**Uncertainty**
$X = A \pm B$	$\Delta X = \sqrt{(\Delta A)^2 + (\Delta B)^2}$
$X = AB$ or $X = \frac{A}{B}$	$\frac{\Delta X}{X} = \sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2}$
$X = A^n$	$\Delta X = n(\Delta A)$ (ΔA & ΔX are %.)
gradient of line	$\Delta m_{\text{dataline}} = \frac{\lvert m_2 - m_1\rvert}{\sqrt{n-2}}$
y-intercept	$\Delta c = \frac{\lvert c_2 - c_1\rvert}{\sqrt{n-2}}$

8 Mathematical & Physical Constants

See Table 5.

Table 3 Mathematical Notation

$\in$	'is an element of'
$\notin$	'is not an element of'
$\subset$	'is a strict subset of'
$\subseteq$	'is a subset of'
$\cup$	union
$\cap$	intersection
$[a,b]$	$x \in \mathbb{R} : a \leq x \leq b$
(a,b)	$x \in \mathbb{R} : a < x < b$
$[a,b)$	$x \in \mathbb{R} : a \leq x < b$
$\forall$	'for all'
$\exists$	'there exists'
$\Rightarrow$	'implies'
$\Leftrightarrow$	'implies and is implied by' / 'is logically equivalent to'
$\therefore$	'therefore'
$\because$	'because'
$\neg$	negation
$\mathbb{N}$	The set of natural numbers
$\mathbb{W}$	The set of whole numbers
$\mathbb{Z}$	The set of integers
$\mathbb{Q}$	The set of rational numbers
$\mathbb{R}$	The set of real numbers
$\mathbb{C}$	The set of complex numbers
$\emptyset$	The empty set
$\cdot$	Scalar Product (vectors and scalars)
$\wedge$	Vector Product (or Cross Product)
$\sum$	Sum
∞	Infinity
i	$\sqrt{-1}$

Table 4 Greek Alphabet

A	alpha	α	I	iota	ι	P	rho	ρ
B	beta	β	K	kappa	κ	Σ	sigma	σ
Γ	gamma	γ	Λ	lambda	λ	T	tau	τ
Δ	delta	δ	M	mu	μ	Υ	upsilon	υ
E	epsilon	ϵ	N	nu	ν	Φ	phi	ϕ
Z	zeta	ζ	Ξ	xi	ξ	X	chi	χ
H	eta	η	O	omicron	ø	Ψ	psi	ψ
Θ	theta	θ	Π	pi	π	Ω	omega	ω

Table 5 Mathematical and Physical Constants and Values

Quantity	**Symbol**	**Value**
Acceleration due to gravity on Earth	g	9.8ms^{-2}
Universal Gravitational Constant	G	$6.67 \times 10^{-11}\text{m}^3\text{kg}^{-1}\text{s}^{-2}$
Speed of Light in Vacuum	c	$3 \times 10^8\text{ms}^{-1}$
Speed of Sound in Air	—	$3.4 \times 10^2\text{ms}^{-1}$
Charge on Electron (Magnitude)	e	$1.60 \times 10^{-19}\text{C}$
Mass of Electron	m_e	$9.11 \times 10^{-31}\text{kg}$
Mass of Proton	m_p	$1.673 \times 10^{-27}\text{kg}$
Mass of Neutron	m_n	$1.675 \times 10^{-27}\text{kg}$
Plank's Constant	h	$6.63 \times 10^{-34}\text{Js}$
$h/2\pi$ (Dirac's Constant)	$\hbar$	$1.0546 \times 10^{-31}\text{Js}$
Permeability of Free Space	μ_0	$4\pi \times 10^{-7}\text{Hm}^{-1}$
Permittivity of Free Space	ϵ_0	$8.85 \times 10^{-12}\text{Fm}^{-1}$
Radius of Earth	—	$6.4 \times 10^6\text{m}$
Mass of Earth	—	$6 \times 10^{24}\text{kg}$
Radius of Moon's Orbit	—	$3.84 \times 10^8\text{m}$
Mass of Moon	—	$7.3 \times 10^{22}\text{kg}$
Pi	π	3.141592
Euler's Number	e	2.718282
Square root of two	$\sqrt{2}$	1.414214