Unit Assessment – (Advanced Higher) Mathematics 2

Outcome 1 *Marks*

1. Find the derivative of the function, *f,* defined by

$$f(x) = \tan^{-1}(4x)$$ 2

2. Use implicit differentiation to find an expression for $\frac{dy}{dx}$ for the hyperbola

$$4x^2 - 6y^2 = 7$$ 3

3. A curve is given by the parametric equations $x = \cos t$, $y = \sin t$.

Find $\frac{dy}{dx}$. 2

Outcome 2

4. Find $\int \frac{2x-4}{x(x-1)}dx$. 3

5. Use the method of integration by parts to evaluate $\int_1^2 2xe^x dx$. 4

6. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{\sin x}{y}$. 2

Outcome 3

7. $z = 1+4i$ and $w = 1-2i$ are two complex numbers.

a) Express *zw* in Cartesian form and plot *zw* on an Argand diagram. 2
b) Find the modulus and argument of *w* and hence write *w* in polar form. 3

Outcome 4

8. For the arithmetic sequence 200, 175, 150, … find:

a) The 20th term. 2
b) The sum of the first 20 terms. 2

9. For the geometric sequence 5, 30, 180, … find:

a) The 10th term. 2
b) An expression for the sum of the first n terms. 2

Outcome 5

10. For any real numbers a and b, it is conjectured that

$$a^2 > b^2 \Rightarrow a > b$$

Disprove this conjecture by providing a counter-example. 2

11. Prove, by using proof by contradiction, that $\sqrt{2}$ is irrational. 3

Unit Assessment – (Advanced Higher) Mathematics 2 Solutions

1.

$$f(x) = \tan^{-1}(4x)$$

$$f'(x) = \frac{1}{1+(4x)^2}.4 = \frac{4}{1+16x^2}$$

2.

$$4x^2 - 6y^2 = 7$$

$$\Rightarrow \frac{d}{dx}(4x^2 - 6y^2) = \frac{d}{dx}(7)$$

$$8x - 12y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{2x}{3y}$$

3.

$$x = \cos t \qquad y = \sin t$$

$$\frac{dx}{dt} = -\sin t \qquad \frac{dy}{dt} = \cos t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \frac{-\cos t}{\sin t} = -\cot t$$

4.

$$\int \frac{2x-4}{x(x-1)}dx$$

$$\text{Partial Fractions}: \frac{2x-4}{x(x-1)} = \frac{A}{x} + \frac{B}{(x-1)}$$

$$= \frac{A(x-1)+B(x)}{x(x-1)}$$

$$2x - 4 = A(x-1) + B(x) \Rightarrow A = 4 \Rightarrow B = -2$$

$$\int \frac{2x-4}{x(x-1)}dx = \int \frac{4}{x}dx + \int \frac{-2}{x-1}dx$$

$$= 4\ln|x| - 2\ln|x-1| + C$$

5.

$$\int_1^2 2xe^x dx$$

Let $f = 2x$ then $f' = 2$; $g' = e^x$ then $g = e^x$

$$\int fg' = fg - \int f'g$$

$$= \left[2xe^x\right]_0^2 - \int_0^2 2e^x$$

$$= \left[(4e^2)\right] - \left[2e^x\right]_0^2$$

$$= 4e^2 - \left[2e^2 - 2\right]$$

$$= 2e^2 + 2$$

6.

$$\frac{dy}{dx} = \frac{\sec^2 x}{y} \Rightarrow \int y dy = \int \sec^2 x dx$$

$$\frac{y^2}{2} = \tan x + C$$

$$y = \pm\sqrt{2\tan x + k}$$

7.

a)

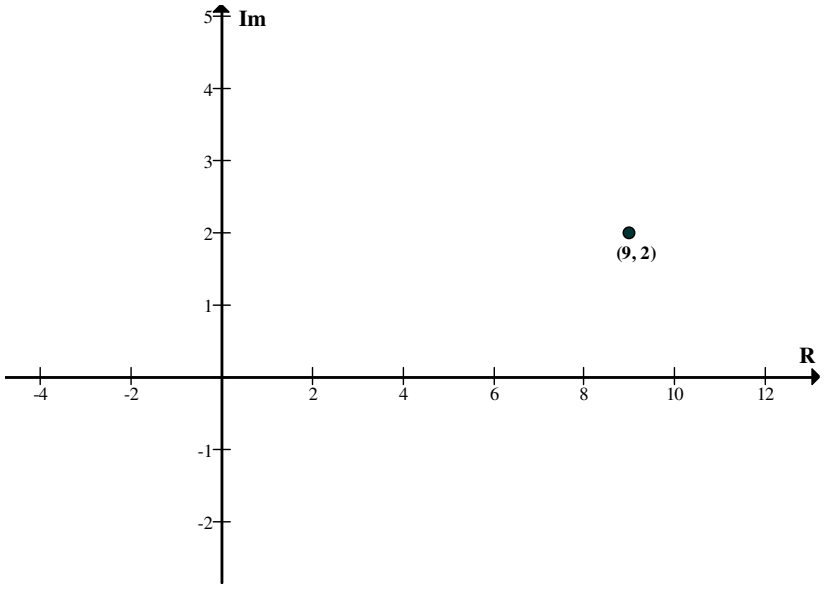
$z = 1 + 4i \qquad w = 1 - 2i$

$$zw = (1+4i)(1-2i)$$

$$= 1 + 4i - 2i - 8i^2$$

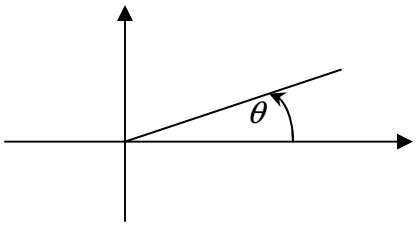
$$= 1 + 2i - 8(-1)$$

$$= 9 + 2i$$



b)

$|z| = \sqrt{1^2 + 4^2} = \sqrt{17}$



$\theta = \tan^{-1} 4 = 76°$

$\Rightarrow \sqrt{17}(\cos 76° + i \sin 76°)$

8.

a)

$d = -25$

$a = 10$

$U_n = a + (n-1)$

$U_{20} = 100 + (20-1) - 25 = -375$

b)

$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = 10\left[2 \times 100 + 19(-25)\right] = -2750$

9.

a)

$r = 6$

$a = 5$

$U_{10} = ar^{n-1} = 5 \times 6^9 = 50388480$

b)

$S_n = \frac{a(1-r^n)}{1-r} = \frac{5(1-6^n)}{1-6} = -(1-6^n)$

10.

Let $a = -2$ and $b = 1$

$(-2)^2 > (1)^2$

$4 > 1$

$\therefore\ a^2 > b^2$ but $b > a$

11.

Assume that $\sqrt{2}$ is rational. Let $\sqrt{2} = \frac{p}{q}$ where p and q have no common factors and are positive integers.

$\therefore\ 2q^2 = p^2 \Rightarrow p^2$ is even. $\Rightarrow p$ is even

Let $p = 2m$ for an integer m $\therefore\ 2q^2 = (2m)^2 = 4m^2 \Rightarrow q^2 = 2m^2$

$q^2 = 2m^2 \Rightarrow q^2$ is even and so q is even .

The original assumption that $\sqrt{2}$ is rational is false since p and q are even but were assumed to have no common factors. The original conjecture that $\sqrt{2}$ is irrational is true.