Unit assessment – (Advanced Higher) Mathematics 1

Outcome 1

Marks

1. Expand $(2x+y)^4$ — 2
2. Express $\dfrac{5x+7}{(x+3)(x-1)}$ in partial fractions. — 3

Outcome 2

3. Differentiate with respect to *x:*

   a) $f(x) = 9x^5 \ln x$ — 2

   b) $f(x) = \dfrac{7x+9}{2x-6}$ — 2

   c) $f(x) = \exp(6x + \tan x)$ — 2

Outcome 3

4. Find:

   a) $\displaystyle\int \frac{3x^2}{x^3-15}dx$ — 2

   b) $\displaystyle\int e^{6x}dx$ — 2

5. Find $\int \cos^4 x \sin x \, dx$, using $u = \cos x$ — 3

Outcome 4

6. $y = \dfrac{x^2+3x+3}{x+1}$

   a) Find the equation of the vertical asymptote. — 1

   b) Show that the graph has a non-vertical asymptote and find its equation. — 2

   c) Sketch *y=f(x)* showing intersections with the axes and also turning points with justification. — 6

Outcome 5

7. Using Gaussian Elimination, solve the following:

$$\begin{aligned} x + y + z &= 3 \\ x + 5y + 2z &= 2 \\ 2y - z &= 4 \end{aligned}$$

5

Unit assessment – (Advanced Higher) Mathematics 1 Solutions

1.

$$(2x+y)^4 = \binom{4}{0}(2x)^4 + \binom{4}{1}(2x)^3 y + \binom{4}{2}(2x)^2 y^2 + \binom{4}{3}(2x) y^3 + \binom{4}{4} y^4$$
$$= (1)16x^4 + (4)(8x^3)y + (6)(4x^2)y^2 + (4)(2x)y^3 + (1)y^4$$
$$= 16x^4 + 32x^3 y + 24x^2 y^2 + 8xy^3 + y^4$$

2.

$$\frac{5x+7}{(x+3)(x-1)} = \frac{A}{(x+3)} + \frac{B}{(x-1)}$$
$$= \frac{A(x-1)+B(x+3)}{(x+3)(x-1)}$$

$5x+7 = A(x-1)+B(x+3)$

Let $x=1 \Rightarrow 12 = 4B \Rightarrow B = 3$

Let $x=-3 \Rightarrow A = 2$

$$\therefore \frac{5x+7}{(x+3)(x-1)} = \frac{2}{(x+3)} + \frac{3}{(x-1)}$$

3.

a) $f(x) = 9x^5 \ln x$

Let $u = 9x^5,\ u' = 45x^4$ and $v = \ln x,\ v' = \frac{1}{x}$

$$f'(x) = u'v + uv' = 45x^4 \ln x + \frac{9x^5}{x}$$
$$= 9x^4(5\ln x + 1)$$

b)

$u = 7x+9$ then $u' = 7$

$v = 2x-6$ then $v' = 2$

$$f'(x) = \frac{u'v - uv'}{v^2}$$
$$f'(x) = \frac{7(2x-6) - 2(7x+9)}{(2x-6)^2} = \frac{-60}{(2x-6)^2}$$

c)

$$f(x) = \exp(6x + \tan x)$$
$$f'(x) = 6 + \sec^2 x.e^{(6x+\tan x)}$$

4.

a) $\int \frac{3x^2}{x^3 - 15} dx = \ln|x^3 - 15| + C$

b) $\int e^{6x} dx = \frac{e^{6x}}{6} + C$

5.

Let $u = \cos x \;\Rightarrow\; \frac{du}{dx} = -\sin x$

Rearranging: $dx = \frac{du}{-\sin x}$

$$\begin{aligned}\therefore \int \cos^4 x \sin x \, dx &= \int \frac{u^4 \sin x}{-\sin x} du \\ &= \int -u^4 du \\ &= \frac{-u^5}{5} + C \\ &= \frac{-\cos^5 x}{5} + C\end{aligned}$$

6.

a) $x = -1$

b) The function $y = \frac{x^2 + 3x + 3}{x + 1}$ is improper so divide through by the denominator:

$$\begin{array}{r|l} & x + 2 \\ \hline x+1 & x^2 + 3x + 3 \\ & x^2 + x \\ \hline & \quad 2x + 3 \\ & \quad 2x + 2 \\ \hline & \qquad 1 \end{array}$$

$$\Rightarrow \quad y = x + 2 + \frac{1}{x+1}$$

As $x \to \pm\infty,\ y \to x+2$. Therefore, there is a slant (non-vertical) asymptote at $y = x+2$.

c)

X – axis crossing: Let $y = 0 \Rightarrow x^2 + 3x + 3 = 0$

Graph does not cross x-axis as there are no real roots.

Y - axis crossing: Let $x = 0$. The graph crosses the y-axis at (0, 3)

Turning points:

$$y = \frac{x^2+3x+3}{x+1} = x+2+\frac{1}{x+1}$$

$$\frac{dy}{dx} = 1 - \frac{1}{(x+1)^2} = 0 \text{ for S.P}$$

$$\Rightarrow (x+1)^2 = 1 \Rightarrow x+1 = \pm 1 \Rightarrow x = 0,\ -2$$

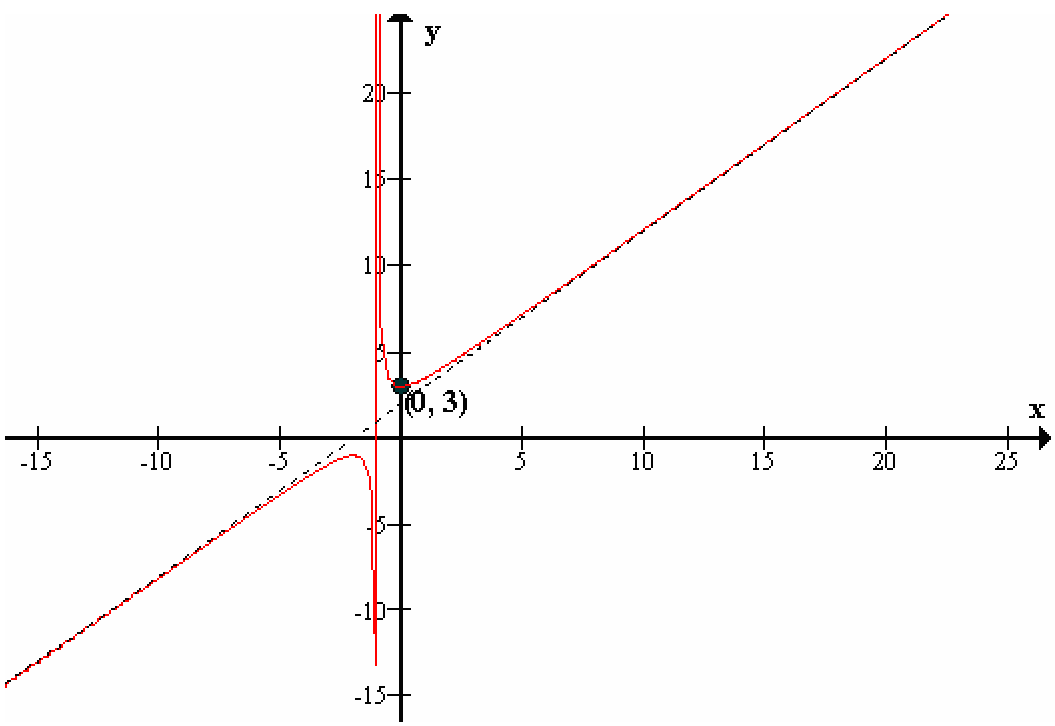
$$\therefore\ S.P\ are\ (0,\ 3),\ (\text{-}2,\ \text{-}1)$$

Nature of T.P:

$$\frac{d^2y}{dx^2} = -\frac{1}{(x+1)^3}$$

$y''(0) = -1 < 0$. Therefore (0, 3) is a max t.p.

$y''(-2) = 1 > 0$. Therefore (-2, -1) is a min t.p.



7.

$$\begin{aligned} x + y + z &= 3 \\ x + 5y + 2z &= 2 \\ 2y - z &= 4 \end{aligned}$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 5 & 2 & 2 \\ 0 & 2 & -1 & 4 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -4 & -1 & 1 \\ 0 & 2 & -1 & 4 \end{array}\right) r_1 - r_2$$

$$= \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -4 & -1 & 1 \\ 0 & 0 & -3 & 9 \end{array}\right) r_2 + 2r_3$$

$$\Rightarrow (x, y, z) = (5.5, \frac{5}{2}, -3)$$