Unit 2

Polynomials and Quadratics

- Polynomials are expressions in the form $3x^4+2x^3-6x^2+11$
- Polynomials higher than quadratics must be factorised using:

Synthetic Division

eg $5x^4+2x^2+8x-2$ by $(x-2)$

2	5	2	8	−2
		10	24	64
	5	12	32	62

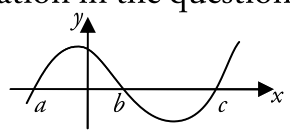
soln: $(x-2)(5x^2+12x+32)+62$

$(x-2)$ – divisor; $(5x^2+12x+32)$ – quotient; 62 – remainder

- If the remainder is 0 then the divisor is a factor
- In expressions like $5x^3+3x-4$ where there is a missing term, eg x^2, a zero must be placed on the top line of the table in the correct place, eg
- Similarly, if a letter is a coefficient, eg $4x^3+px^2+3x+1$ then a letter must be placed in the table. An equation can then be formed using the remainder and information in the question.

5	0	3	−4

Equations of Polynomials from a Graph



- If a graph is given and the roots are known, eg then the equation can be calculated using $f(x)=k(x-a)(x-b)(x-c)$

Curve Sketching

- Synthetic division is common in these questions. See U1OC3.

Iteration

- Used if a root has to be calculated to a given number of decimal places.

Quadratics

- In $y=ax^2+bx+c$ if $a>0$ then $\cup$ – shaped and stat. pt. is a min.
  if $a<0$ then $\cap$ – shaped and stat. pt. is a max.

Sketching a Quadratic

- Identify shape, ie $\cup$ or $\cap$
- Find y-axis intercept $(x=0)$
- Find roots $(y=0)$
- Axis of symmetry
- Coords of turning point

Completing the Square

- The coefficient of x^2 must be 1 before starting. This may mean taking out a common factor from x^2 and x eg $2x^2-4x+3=2(x^2-2x)+3$
- When $y=ax^2+bx+c$ is written in the form $y=a(x+p)^2+q$ the turning point is $(-p,q)$
- In this example, the turning point is $(3,-2)$

Example

x^2-6x+7

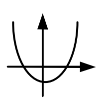
$=\left[x^2-6x+(-3)^2-(-3)^2\right]+7$

$=\left[(x-3)^2-9\right]+7$

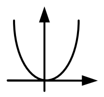
$=(x-3)^2-9+7$

$=(x-3)^2-2$

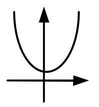
The Discriminant



$b^2-4ac>0$ two roots, real and unequal



$b^2-4ac=0$ one root, real and equal



$b^2-4ac<0$ no roots, not real

Intersection Between a Line and Curve

Substitute (or equate) equation of line into curve then use discriminant to show two, one (tangent) or no points of intersection.

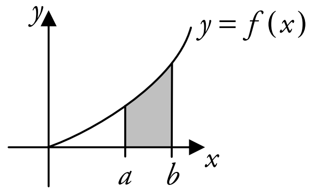
Integration

- $\int ax^n\,dx=\dfrac{ax^{n+1}}{n+1}+C \quad n\neq -1$

  "raise the power by one, divide by the new power then add C"

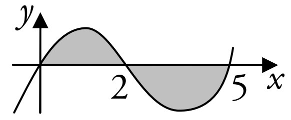
- The area between the graph $y=f(x)$ and the x-axis from $x=a$ to $x=b$ is $\int_a^b f(x)\,dx$ note: no "$+C$"

  Integrate, then sub in (top minus bottom)



- If the area is split by the x-axis then calculate as 2 separate areas, eg

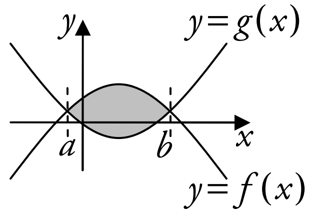
  $\int_0^2 f(x)\,dx$ and $\int_2^5 f(x)\,dx$ ← Gives negative area. Ignore sign and add the two



- Area enclosed between two graphs $y=f(x)$ and $y=g(x)$ from $x=a$ to $x=b$ is given by

  $\int_a^b f(x)-g(x)\,dx,\ f(x)\geq g(x)$

  "integrate top curve minus bottom curve"



- If a and b are not know, they can be calculated by equating two equations.

The Circle

- The equation of a circle with centre (a,b) and radius r is $(x-a)^2+(y-b)^2=r^2$
- General equation: $x^2+y^2+2gx+2fy+c=0$; centre is $(-g,-f)$, radius is $\sqrt{g^2+f^2-c}$
- All of the above is given in the exam
- A circle and line can have two, one (tangent) or no points of intersection. To work this out, substitute equation of line into circle and solve (ie factorise). For tangency, $b^2-4ac=0$ can be used.
- A tangent is a straight line, and $y-b=m(x-a)$ gives its equation. (a,b) will be the point given, and since the tangent is perpendicular to the line from the centre of the circle, use $m_1\times m_2=-1$ to find its gradient

Trigonometry

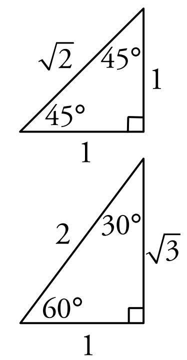
Radians

- Radians → Degrees: replace π by 180°, simplify. eg $\frac{\pi}{2}=\frac{180°}{2}=90°$
- Degrees → Radians: ÷ by 180, × by π, simplify. eg $45°=\frac{45\pi}{180}=\frac{\pi}{4}$
- π radians $=180°$

$2\pi=360°$ $\quad\frac{\pi}{2}=90°$ $\quad\frac{\pi}{3}=60°$

$\frac{\pi}{4}=45°$ $\quad\frac{\pi}{6}=30°$ $\quad\frac{3\pi}{4}=135°$

Exact Values



Algebraic Solutions of Trigonometric Equations

- Must use a CAST diagram
- Check inequality to see if answer is in degrees or radians and how many solutions are required eg $0\leq x\leq\pi$ or $0\leq x\leq 360°$
- Remember for $\sin\underline{3}x=0.5,\ 0\leq x\leq 360°$, there will be 3 pairs of solutions

3D Trigonometry

Questions in 3D are dealt with in the same way as 2D – usually, SOH CAH TOA or sine/cosine rule is used

Must Know

- Pythagoras's Theorem
- SOH CAH TOA
- Sine Rule: $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$
- Cosine Rule: $a^2=b^2+c^2-2bc\cos A$ or $\cos A=\dfrac{b^2+c^2-a^2}{2bc}$
- $\sin^2 x+\cos^2 x=1 \quad\Rightarrow \sin^2 x=1-\cos^2 x$
  $\quad\Rightarrow \cos^2 x=1-\sin^2 x$

Given in Exam

- $\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$
- $\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$
- $\sin 2\alpha=2\sin\alpha\cos\alpha$
- $\cos 2\alpha=\cos^2\alpha-\sin^2\alpha$
  $=2\cos^2\alpha-1$
  $=1-2\sin^2\alpha$
- Remember: if equations involve a $\sin 2\alpha$ with a $\sin\alpha$, or a $\cos 2\alpha$ with a $\cos\alpha$, it must be substituted using one of the above formulae, before attempting to solve.