Unit 1

Straight Lines

Distance Formula

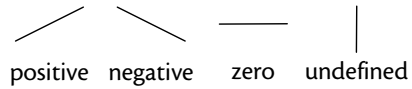
- Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ between (x_1, y_1) and (x_2, y_2)

Midpoint Formula

- Midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

Gradients

- $m = \frac{y_2 - y_1}{x_2 - x_1}$
- $m = \tan\theta$



- Parallel lines have equal gradients

Perpendicular Lines

- $m_1 \times m_2 = -1$ for two lines at right angles, with gradients m_1 and m_2

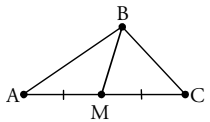
Equation of a Straight Line

- $y - b = m(x - a)$
- (a, b) is a point on the line
- m is the line's gradient

Intersection of two lines

- Solve simultaneous equations (the equations of the two lines)
- Elimination, equating or substitution

Medians

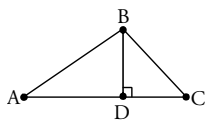


- BM is a median
- M is the midpoint of AC

Median from B

- Point: B or M
- Gradient: m_{BM}

Altitudes



- BD is an altitude
- BD is perp. to AC

Altitude from B

- Point: B
- Gradient: m_{BD}

$m_{BD} \times m_{AC} = -1$

Functions and Graphs

Notation

- $\in$ – belongs to
- $\mathbb{N}$ – natural numbers $\{1, 2, 3, ...\}$
- $\mathbb{W}$ – whole numbers $\{0, 1, 2, 3, ...\}$
- $\mathbb{Z}$ – integers $\{..., -1, 0, 1, 2, ...\}$
- $\mathbb{Q}$ – rational – integer fractions
- $\mathbb{R}$ – real – all points on the number line

Composite Functions

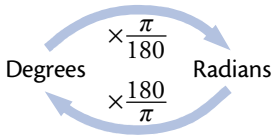
Example

$f(x) = x^2 + 1$, $g(x) = 3x - 4$

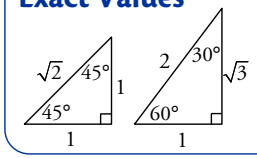
$f(g(x))$	$g(f(x))$
$= f(3x-4)$	$= g(x^2+1)$
$= (3x-4)^2 + 1$	$= 3(x^2+1) - 4$

Radians

- π radians $= 180°$

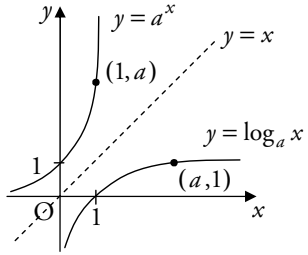


Exact Values



Graphs of Inverses

- Reflect in the line $y = x$, eg logarithmic and exponential



Domains

- Set of numbers a function can operate on

Restrictions

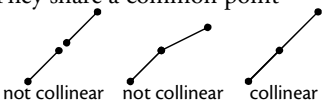
- Cannot divide by zero
- Cannot take even roots (eg square root) of negative numbers

Graph Transformations

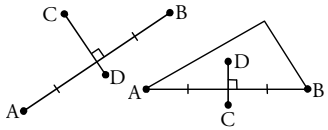
- Translation: $f(x)+a$, $f(x+a)$
- Reflection: $-f(x)$, $f(-x)$
- Scaling: $kf(x)$, $f(kx)$

Collinearity

- The gradient between two of the points is equal to the gradient between two other points
- They share a common point



Perpendicular Bisectors



- CD is a perpendicular bisector of AB
- CD passes through the midpoint of AB

Perpendicular bisector of AB

- Point: midpoint of AB
- Gradient: m_{CD}

$m_{CD} \times m_{AB} = -1$

Sequences

- A linear recurrence relation can be written in the form

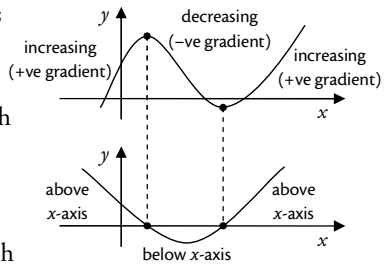
$u_{n+1} = au_n + b$ or $u_n = au_{n-1} + b$

(both mean exactly the same thing)

- If $-1 < a < 1$ then a linear recurrence relation will have a limit l, which can be calculated using $l = \frac{b}{1-a}$

Graphs of Derived Functions

- All stationary points become roots
- When the graph is **increasing**, the graph of the derivative is **above** the x-axis
- When the graph is **decreasing**, the graph of the derivative is **below** the x-axis



Differentiation

Rules

- If $f(x) = ax^n$ then $f'(x) = anx^{n-1}$

"power multiplies to the front, power lowers by one"

Preparing to differentiate

- Multiply out brackets
- Change roots into powers, eg $\sqrt{x} = x^{\frac{1}{2}}$
- Put all x terms on the top line, eg $\frac{3}{x^2} = 3x^{-2}$ and $\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

Rates of Change

- $f'(a)$ is the rate of change of $f(x)$ at $x = a$.

Equations of Tangents

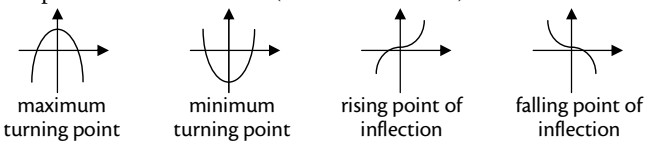
- A tangent is a straight line – to use $y - b = m(x - a)$ we need a point and a gradient
- One coordinate of the point will always be given – the other can be worked out using the given equation
- The gradient is a rate of change – differentiate and substitute in x-coord. ($\frac{dy}{dx}$ is the equation of the gradient)

Increasing and Decreasing Functions

- $\frac{dy}{dx} > 0$ – increasing, $\frac{dy}{dx} < 0$ – decreasing

Stationary Points

- Occur when $f'(x) = 0$
- Four possibilities for nature (use a nature table):



Curve sketching

- Work out x-axis intercepts (roots) – solve $y = 0$
- Work out y-axis intercept – find y for $x = 0$
- Determine stationary points and their nature (use a nature table)

Closed Intervals (Restricted Domains)

- Max/min values can occur at stationary points, or end points of the closed interval

Optimisation

- Problems usually involve finding max/min areas and volumes
- Calculate a stationary point ($f'(x) = 0$) and determine its nature (use a nature table)